Math 150

calculus

تمارين على جميع الوحدات في الكتاب

ريض ١٥٠

إعداد : محمد الرفاعي


٠٥٥٧٥٤٨٧٣٦


judimhmdrefaa@


judimhmdrefaa@

Chapter (1)

Section 1.1

Math 150 standards

Q1: Guess the value of the indicated limit by completing the given table.

$$\lim_{x\to 1} f(x), \text{ where } f(x) = \frac{6x-6}{x^3-1}.$$

x	0.9	0.99	0.999	0.9999	0.99999
$f(x)$					
x	1.1	1.01	1.001	1.0001	1.00001
$f(x)$					

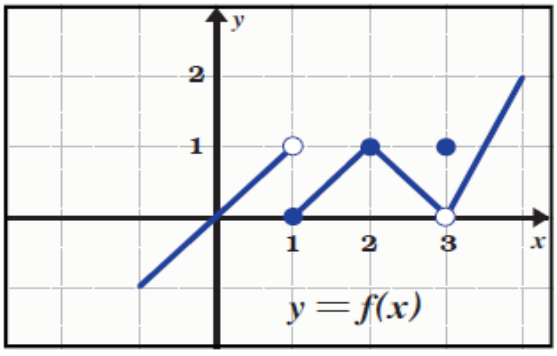
Q3: For the function f graphed in the given Figures, find the following.

a. $f(1)$

b. $\lim_{x\to 1} f(x)$

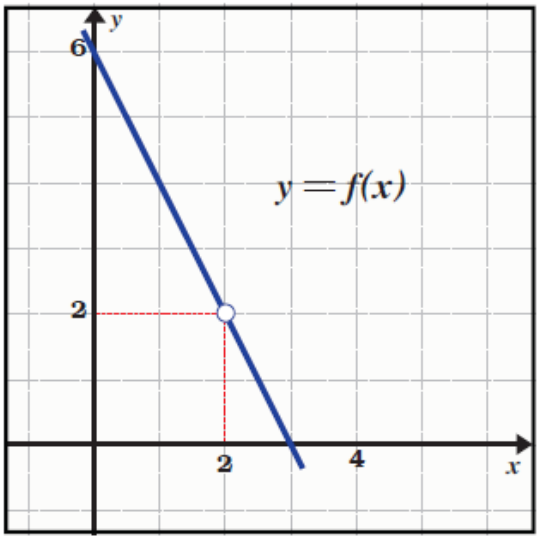
c. $\lim_{x\to 2} f(x)$

d. $\lim_{x\to 3} f(x)$



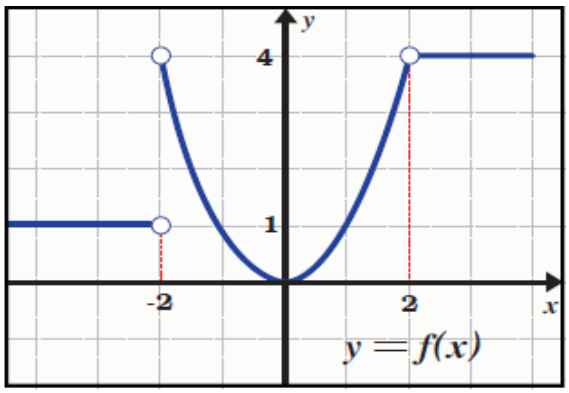
a. $f(2)$ b. $\lim_{x\to 2^+} f(x)$

c. $\lim_{x\to 2^-} f(x)$ d. $\lim_{x\to 2} f(x)$



a. $\lim_{x\to 2} f(x)$ b. $\lim_{x\to -2^+} f(x)$ c. $\lim_{x\to -2^-} f(x)$

d. $\lim_{x\to 0^-} f(x)$ e. $\lim_{x\to 0^+} f(x)$ f. $\lim_{x\to 0} f(x)$



Math 150 standards

Q4: For the given functions, compute the following limits

$$f(x)=\begin{cases}x^3+2x+1, & x<1\\ 3x-1, & x\geq 1\end{cases}$$

1 $\lim_{x\to 2} f(x)$

2 $\lim_{x\to 1} f(x)$

$$f(x)=\begin{cases}1 & \text{if } 0\leq x<1\\ x^2 & \text{if } x>1\end{cases}$$

1 $\lim_{x\to 1+} f(x)$

2 $\lim_{x\to 1-} f(x)$

Q5: Evaluate the limit if exists.

1 $\lim_{x\to 2}(x^2-6x+1)$

2 $\lim_{x\to 1}\frac{3x^3-8}{x-2}$

3 $\lim_{x\to 0}(|x|-|-3|)$

4 $\lim_{x\to 2}\frac{x^4-16}{x^2-4}$

Q5: Evaluate the limit if exists.

5 $\lim_{x \to -2^-} \sqrt[3]{x-6}$

6 $\lim_{t \to 9} \frac{9-t}{3-\sqrt{t}}$

7 $\lim_{x \to 0} \frac{3x - |x|}{|2x| - 5x}$

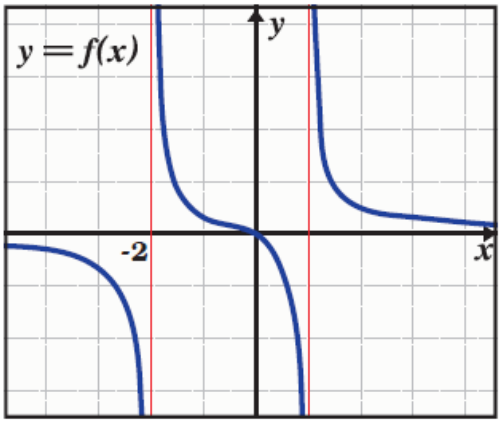
8 $\lim_{x \to 1} \frac{x^2+4x-5}{x^2-1}$

9 $\lim_{x \to 0} \frac{\sqrt{x^2+9}-3}{x^2}$

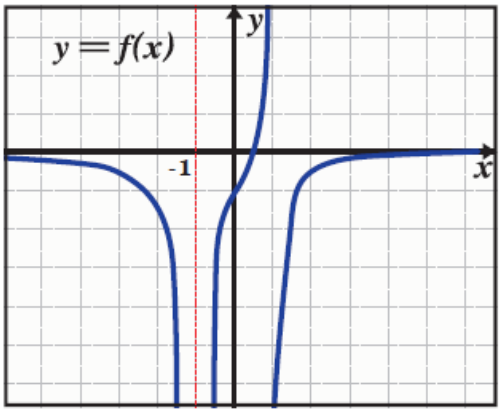
Math 150 standards

Q6: Consider the figures to determine the following limit if exists.

1 $\lim_{x \to -2} f(x)$



2 $\lim_{x \to -1} f(x)$



Q7: Find the vertical asymptotes (if any) of the graph of the given functions

1 $f(x) = \dfrac{x-1}{x^2+1}$

2 $f(x) = \dfrac{2+x}{x^2(1-x)}$

3 $f(x) = \dfrac{1}{x^2-4}$

4 $f(x) = \dfrac{x^2+1}{(x-1)^3}$

Q8: Find the following limits

1 $\lim_{x \to 5+} \frac{6}{x-5}$

2 $\lim_{x \to -2-} \frac{x-1}{x^2(x+2)}$

3 $\lim_{x \to 3-} \frac{1}{(x-3)^8}$

4 $\lim_{x \to 1} \frac{x^2+3x+2}{(x-1)^2}$

5 $\lim_{x \to 0} \frac{1}{x^2}$

Q9: Find the given limit, if exists.

1 $\lim_{x\to\infty}\frac{5x^2-3x+1}{2x^2+4x-7}$

2 $\lim_{x\to-\infty}\frac{4-7x}{2+3x}$

3 $\lim_{x\to-\infty}\frac{2-x^2}{x+3}$

4 $\lim_{x\to\infty} x-\sqrt{x^2+7}$

5 $\lim_{x\to-\infty}\frac{4x-3}{\sqrt{x^2+1}}$

6 $\lim_{x\to\infty}\frac{-2x+1}{\sqrt{x^2+x}}$

Q10: Find the horizontal asymptotes (if any) of the given functions

1 $f(t)=\frac{t-1}{t^2+1}$

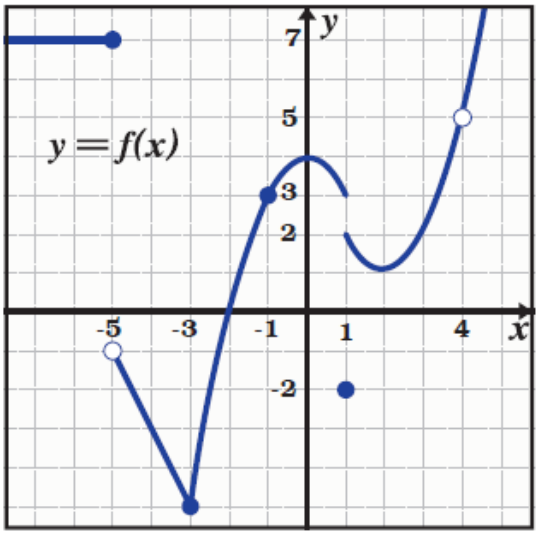
2 $f(x)=\frac{-4x^2}{x^2+4}$

3 $f(x)=\frac{2}{4-\frac{7}{x^2}}$

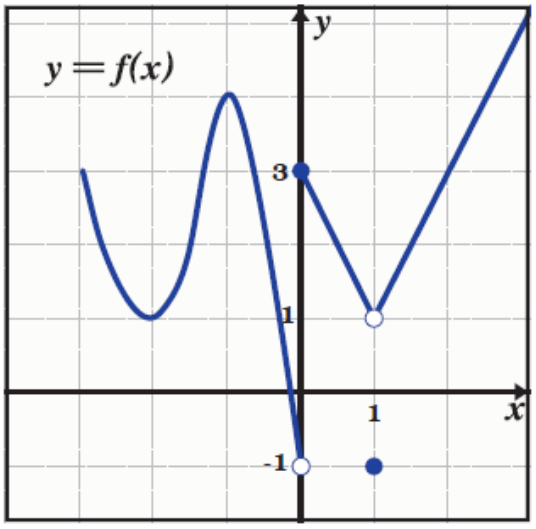
Math 150 standards

Q11: Use the given figure to discuss the continuity of the function at

$x = -5, -3, -1, 1, 4$.



Q12: Use the given graph of the function $f(x)$ to find all number(s) at which $f(x)$ is discontinuous



Q13: Discuss the continuity of each function at the given number

1 $f(x) = -\dfrac{1}{(x-1)^2}$, $x = 1$

2 $f(x) = \dfrac{x+1}{x-2}$, $x = 3$

3 $f(x) = \begin{cases} x+1 & \text{if } x \le 2 \\ 2 & \text{if } x > 2 \end{cases}$, $x = 2$

4 $f(x) = \begin{cases} \dfrac{x^2-9}{x-3} & \text{if } x \ne 3 \\ 4 & \text{if } x = 3 \end{cases}$, $x = 3$

5 $f(x) = \begin{cases} 5x & \text{if } x < 1 \\ 5 & \text{if } x = 1 \\ x+4 & \text{if } x > 1 \end{cases}$, $x = 1$

6 $f(x) = \begin{cases} \dfrac{|x-3|}{x-3} & \text{if } x \ne 3 \\ 1 & \text{if } x = 3 \end{cases}$, $x = 3$

Q14: Find the points of discontinuity of f

1 $f(x) = \dfrac{x-1}{x^2+x-2}$

2 $f(x) = \dfrac{x-1}{\sqrt{x^2-1}}$

Q15: Find a constants a and b, so that the given function will be continuous for all x

1 $$f(x)=\begin{cases} ax+3 & \text{if } x>5 \\ 8 & \text{if } x=5 \\ x^2+bx+1 & \text{if } x<5 \end{cases}.$$

2 $$f(x)=\begin{cases} ax^2+b & \text{if } x<1 \\ 1 & \text{if } x=1 \\ 2b & \text{if } x>1 \end{cases}$$

Q16: Find the vertical asymptotes, if any, of each of the following functions

1 $$f(x)=\frac{x^2-2}{x^2-x-2}$$

2 $$f(x)=\frac{x^2}{x^2-4}$$

3 $$f(x)=\frac{x}{x^2+x+4}$$

Q17: Use the Intermediate Value Theorem to show that each of the following equations has at least one root in the specified interval.

1 $x^4 - x^2 - 6 = 0,\ [1,2]$

2 $x^7 - x^2 - x - 1 = 0,\ [0,3]$

Q18: Show that $f(x) = x^3 + 2x - 15$ has a zero somewhere in the interval $[2,3]$.

Math 150 standards

Q19: Evaluate the given limit

1 $\lim_{x\to 0} \frac{3}{x \csc x}$

2 $\lim_{x\to 0} \frac{\sin 3x}{\sin 2x}$

3 $\lim_{x\to 0} \frac{\sin 7x}{x}$

4 $\lim_{\theta\to 0} \frac{\sin 2\theta}{\tan \theta}$

5 $\lim_{x\to 0} \frac{x\sqrt{2}}{\sin(x/3)}$

6 $\lim_{x\to 0} \frac{3x}{\tan(3x) + \sin(7x)}$

7 $\lim_{x\to 0} \frac{\tan(5x)}{\sqrt{2x+1} - 1}$

8 $\lim_{x\to 0} \frac{4x^2}{1 - \cos 3x}$

Q20: Use Squeeze Theorem to evaluate the given limit.

1 $\lim_{x\to 0-} x^3 \cos\left(\frac{2}{x}\right)$

2 $\lim_{x\to 0+} \left(\sqrt{x}\sin\left(x+\frac{1}{x}\right)\right)$

Chapter (2)

Math 150 standards

Q21: Use the limit definition of derivatives to find $f'(x)$.

1 $f(x) = 2x + 1$

2 $f(x) = x^2 + 2x$

3 $f(x) = \frac{1}{\sqrt{x}}$

4 $f(x) = \frac{x}{x+2}$

Q22: Find an equation of the line tangent to the graph of $y = f(x)$ at the point $x = a$.

1 $f(x) = \dfrac{8}{x+4}, a = -2$

2 $f(x) = \sqrt{3x}\,;\ x = 3$

3 $f(x) = 2x + 1;\ x = 1$

4 $f(x) = x^2 - 1, a = 1$

Q23: Write the value of the given limit using the derivative notation

1 $\lim\limits_{h \to 0} \dfrac{(2+h)^2 - 4}{h}$

2 $\lim\limits_{h \to 0} \dfrac{\sqrt{9+h} - 3}{h}$

Q24: Find the derivative of the given function

1 $f(x) = 3x^3 - 4x^2 + 5x - 2$

2 $f(x) = \dfrac{x+2}{x-2}$

3 $f(x) = x\sqrt{x}$

4 $f(x) = \dfrac{6-(1/x)}{x-2}$

Q25: Find the equation of the line tangent to the graph of the given function at the specified point

1 $f(x) = 1 + x + x^2 + x^3 ; (1,4)$

2 $f(x) = x^2 - \dfrac{10}{x} ; (-2,9)$

3 $f(x) = 3\sqrt{x} ; (1,3)$

Q26: find $f'(0)$ given that $h(x)$ is a differentiable function at $x = 0$ with $h(0) = 3$ and $h'(0) = 2$. Find the following

1 $f(x) = xh(x)$

2 $f(x) = h(x) - \frac{1}{h(x)}$

3 $f(x) = \frac{x}{h(x)}$

Q27: Suppose that both $f(x)$ and $g(x)$ are differentiable functions at $x = 5$ and suppose that $f(5) = 1$, $f'(5) = 6$, $g(5) = -3$ and $g'(5) = -2$. Find the following

1 $(fg)'(5)$

2 $(f/g)'(5)$

Q28: Suppose that both $f(x)$ and $g(x)$ are differentiable functions at $x=3$, and suppose that $f(3)=4$, $f'(3)=-6$, $g(3)=2$, and $g'(3)=5$. Find the following

1 $(f+g)'(3)$ 2 $(f/g)'(3)$ 3 $(fg)'(3)$

Q29: Evaluate each of the following

1 $\frac{d^2}{dx^2}\left[(1+2x)\frac{d^2}{dx^2}(5-x^3)\right]$ 2 $\frac{d^2}{dx^2}\left[(x^2+1)\frac{d}{dx}\left(\frac{1}{x}\right)\right]$

Q30: Evaluate each of the following

1 $f'''(x)$, where $f(x) = x^3 - 4x + 1$

2 $\frac{d^2f}{dx^2}$, where $f(x) = 3x^{5/2} - x^2$

3 $f''(t)$, where $f(t) = 3t^3 - 4t^2 + \frac{5}{t}$

4 $f^{(5)}(x)$, where $f(x) = 2x^6 - x^3 + 2$

Math 150 standards

Q31: Find $f'(x)$ for each of the following

1 $f(x) = (2x+3)^3$

2 $f(x) = (x^5 - 10)^{30}$

3 $f(x) = \sqrt{1-\sqrt{x}}$

4 $f(x) = (1+x^2)^{-2}$

5 $f(x) = x^2(x+1)^4$

6 $f(x) = \left(x - \frac{2}{x}\right)^3$

Q32: Find $\frac{dy}{dx}$ for each of the following

1 $y = t^2 + 1, \quad t = \sqrt[3]{x}$

2 $y = \frac{1}{u^2}, u = t^2, t = x - 5$

3 $y = 1 + u^2, u = 7t, t = \frac{1}{x}$

Q33: Find $\frac{dy}{dx}$ at $x = 1$

1 $y = t + \frac{1}{t}, t = (3x - 1)^2$

2 $y = \frac{3}{u}, u = t^2, t = \sqrt{x}$

Q34: If $f'(x) = x + \frac{1}{x}$ and $g(x) = \sqrt{x}$. Find $\frac{d}{dx}[(f \circ g)](x)$ for $x > 0$.

Q35: Suppose that $f'(x) = \sqrt{3x + 4}$ and $g(x) = x^2 - 1$. Find $F'(x)$ for $F(x) = f(g(x))$

Q36: Find the second derivative of the given function.

1 $f(x) = (3x - 1)^3$

2 $f(x) = \left(\frac{x}{x-2}\right)^2$

Q37: Find an equation for the line tangent to the graph of f at the indicated point.

1 $f(x) = (2x + 1)^5$,$(0,1)$

2 $f(x) = \sqrt{x^2 + 8x}$,$(1,3)$

Q38: Express the given expression using the prime notation

1 $\frac{d}{dx}[f(\sqrt{x})]$

2 $\frac{d}{dx}[f(x^2 + 2)]$

3 $\frac{d}{dx}[\sqrt{4f(x) + 1}\ ]$

Math 150 standards

Q39: Find $f'(x)$ for each of the following

1 $f(x) = x\csc x$

2 $f(x) = \dfrac{\tan x}{x}$

3 $f(x) = \sqrt{2\sec x}$

4 $f(x) = \cot\left(\dfrac{1}{x}\right)$

5 $f(x) = \tan(\sec x)$

6 $f(x) = \cos^2\left(\dfrac{2}{x}\right)$

Q40: Evaluate the derivative of the given function at the indicated point.

1 $f(x) = \sec\left(\dfrac{x}{2}\right),\ x = \pi/2$

2 $f(x) = \cos(\sin x),\ x = 0$

Q41: Find the second derivative of the given function

1 $f(x) = \cos(5x)$

2 $f(x) = x^2 \sin x$

Q42: Find $\frac{dy}{dx}$.

1 $y = 2u,\ u = \sin t,\ t = x$

2 $y = \frac{1}{u},\ u = \sec t,\ t = x$

Q43: Find an equation of the line tangent to the graph of at the given point

1 $f(x) = 2\cos(x/2),\ (\pi/2, \sqrt{2}\,)$

2 $f(x) = \sin x,\ (\pi/6, 1/2)$

Math 150 standards

Q44: Find the first derivative of the given function.

1 $y = \sqrt{\log_3 x}$

2 $y = (\ln x)^3$

3 $y = e^x \ln x$

4 $y = \tan(\ln x)$

5 $y = \sin(7^x)$

6 $y = \sqrt{3}^{x\sin x}$

7 $y = \log_5(\sqrt{x})$

8 $y = \ln(\sqrt[3]{x})$

9 $y = \sec(e^{x^2})$

10 $y = \cos^2(\ln x)$

11 $y = \ln\left(\frac{(2x+1)^2}{(3x-2)^3}\right)$

Q45: Find an equation of the line tangent to the graph of the given function at the given point

1 $y = e^{-x}$, $(0,1)$

2 $y = \frac{\ln x}{x}$, $(e, 1/e)$

Math 150 standards

Q46: Find $\frac{dy}{dx}$ using implicit differentiation.

1 $x^2 = y^2 + 1$

2 $x^2y + xy^2 = 3x$

3 $\sqrt{1+xy} = y$

4 $\cos(xy) = x$

5 $\ln(1+y^2) = 1 + x$

6 $xe^{xy} = 1$

Q47: Find the slope of the line tangent to the graph of the given curve at the given point

1 $x^2 + y^2 = 1;\ (1/\sqrt{2}, 1/\sqrt{2})$

2 $x^2 - y^2 = 3;\ (2,1)$

Q48: Find the equation of the line tangent to the graph of the given curve at the given point

1 $x^2 + xy + 2y^2 = 28,\ (-2,-3)$

2 $\ln(xy) = x\ ;\ (1,e)$

Q49: Find $\frac{d^2y}{dx^2}$ for each of the following

1 $x + \cos y = y$

2 $xy + x + y^2 = 3$ at $(1,1)$

Q50: Use the logarithmic technique to differentiate the given function

1 $y = (x+1)^x$

2 $y = x^{x^2}$

Math 150 standards

Q51: Verify that the given function satisfies the three hypotheses of Rolle's Theorem over the indicated interval. Then find all possible values of c that satisfy the conclusion of Rolle's Theorem for the given function over the indicated interval.

1 $f(x) = x^2 - 2x - 3;\ [-1, 3]$

2 $f(x) = \frac{x}{3} - \sqrt{x};\ [0, 9]$

3 $f(x) = \cos 2x;\ [0, \pi]$

Q52: Verify that the given function satisfies the hypotheses of Mean Value Theorem over the indicated interval. Then find all possible values of c that satisfy the conclusion of Mean Value Theorem for the given function over the indicated interval.

1 $f(x) = x^2; \quad [1,2]$

2 $f(x) = x(x^2 - x - 2); \ [-1,1]$

Chapter (3)

Math 150 standards

Q53: Evaluate each of the following limits using L'Hôpital's rule.

1 $\lim_{x\to 0}\frac{\sin 2x}{\tan x}$

2 $\lim_{x\to 0}\frac{x+\sin \pi x}{x-\sin \pi x}$

3 $\lim_{x\to 1-}\frac{x\ln x}{1-x}$

4 $\lim_{x\to 0}\frac{e^x-1}{\ln(x+1)}$

5 $\lim_{x\to 0}\frac{3^x-1}{x}$

6 $\lim_{x\to 1}\frac{1-x^3}{1-x^2}$

7 $\lim_{x\to \pi/2}\frac{1-\sin x}{1+\cos 2x}$

8 $\lim_{x\to 0+}\frac{3^{\sin x}-1}{x}$

9 $\lim_{x\to \infty}\frac{\ln(\ln x+1)}{x}$

10 $\lim_{x\to\infty} xe^{-\sqrt{x}}$

11 $\lim_{x\to\infty} \frac{x^2+2x+1}{x^2-5}$

12 $\lim_{x\to 0+}\left(\frac{2}{x}-\frac{1}{\sin x}\right)$

13 $\lim_{x\to\infty}[\sqrt{x+1}-\sqrt{x}]$

14 $\lim_{x\to 0+} \sin x \ln x$

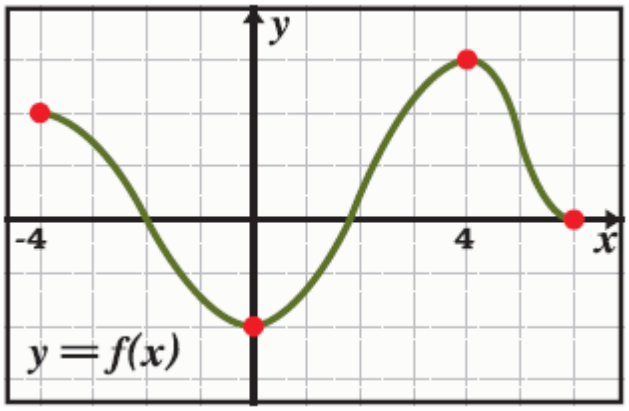
15 $\lim_{x\to-\infty} x^2 e^{3x}$

16 $\lim_{x\to\infty}\left(\frac{1}{x}\ln\frac{1}{x}\right)$

17 $\lim_{x\to\frac{\pi}{2}+}\left(\frac{\pi}{2}-x\right)\tan x$

Math 150 standards

Q54: Use the given Figure which shows the graph of a given function $y = f(x)$

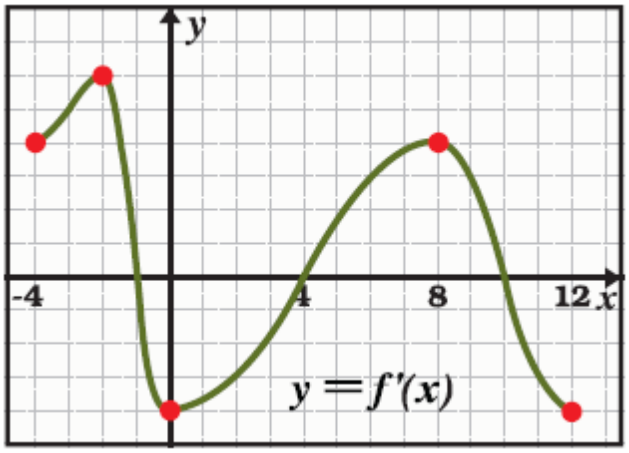


1 Determine the intervals on which f is increasing/decreasing.

2 Find the critical numbers of f.

3 Find the x-coordinates of the points on which f has a relative maximum or a relative minimum.

Q55: Use the given Figure which shows the graph of the first derivative of a given function $y = f(x)$



1 Determine the intervals on which f is increasing/decreasing.

2 Find the critical numbers of f.

3 Find the x-coordinates of the points on which f has a relative maximum or a relative minimum.

Q56: Find the critical numbers of f, the relative extrema of f, and the intervals on which f is increasing or decreasing.

1 $f(x) = x^2 - 6x$

2 $f(x) = x^3 - 6x^2 + 15$

3 $f(x) = \dfrac{3-x}{x+2}$

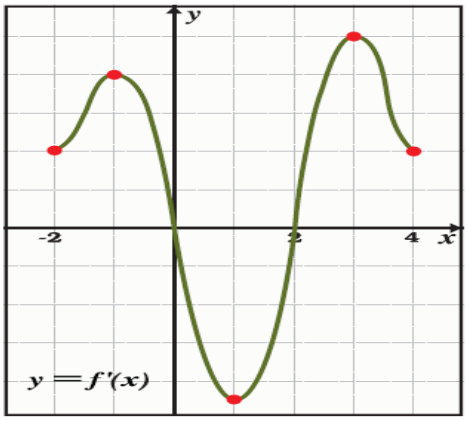
4 $f(x) = \dfrac{x^2}{x^2-9}$

5 $f(x) = 1 + \cos 2x, x \in [0, \pi]$

6 $f(x) = x \ln(2x)$

Math 150 standards

Q57: Use the given Figure which shows the graph of a given function $y = f'(x)$.

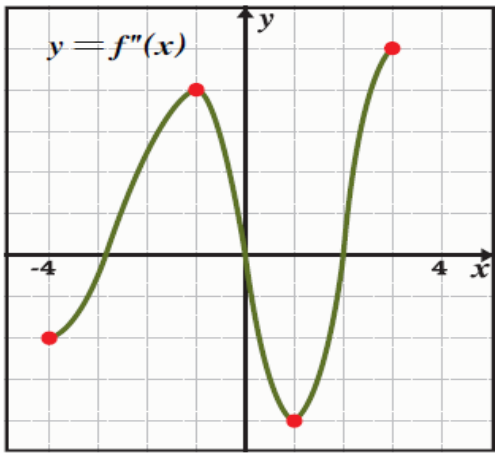


1 Determine the intervals on which f is concave up.

2 Determine the intervals on which f is concave down.

3 Find the $x-$coordinates of all inflection points, if any.

Q58: Use the given Figure which shows the graph of the first derivative of a given function $y = f''(x)$.



1 Determine the intervals on which f is concave up.

2 Determine the intervals on which f is concave down.

3 Find the $x-$coordinates of all inflection points, if any.

Q59: Find the intervals on which f is concave up/down, and find the x-coordinates of the inflection points, if any.

$f(x) = x^3 - 3x$

$f(x) = 2x^3 - 3x^2 - 12x + 5$

$f(x) = (x-1)^3$

$f(x) = x^3(x-4)$

$f(x) = \dfrac{x}{x^2-4}$

$f(x) = \dfrac{x}{x^2+1}$

Math 150 standards

Q60: Find the absolute extrema of the function f, if any

1 $f(x) = \frac{1}{2}x^2 - 2x, [2,5]$

2 $f(x) = 2x^3 + x^2 - 20x + 1, [-3,0]$

3 $f(x) = x^3 - x^2 - 40x + 8, [0,4]$

4 $f(x) = x^4 - 5x^2 + 4, [0,2]$

Math 150 standards

Q61: Sketch the graph of each of the following functions, indicating the relative extrema, concavity, inflection points, and asymptotes, if any.

1 $f(x) = x^2 - 3x - 4$

2 $f(x) = x^3 - 6x^2 + 9x + 6$

3 $f(x) = \frac{x-3}{x+2}$

5 $f(x) = \sqrt{1-x}$

6 $f(x) = x^2 e^{-x}$